Exercise 38

Find y'' by implicit differentiation.

$$x^3 - y^3 = 7$$

Solution

Differentiate both sides with respect to x.

$$\frac{d}{dx}(x^3 - y^3) = \frac{d}{dx}(7)$$

$$\frac{d}{dx}(x^3) - \frac{d}{dx}(y^3) = 0$$

$$(3x^2) - (3y^2) \cdot \frac{d}{dx}(y) = 0$$

$$3x^2 - 3y^2 y' = 0$$

Solve for y'.

$$y' = \frac{x^2}{y^2}$$

Differentiate both sides with respect to x to get y''.

$$\frac{d}{dx}(y') = \frac{d}{dx}\left(\frac{x^2}{y^2}\right)$$

$$y'' = \frac{\left[\frac{d}{dx}(x^2)\right] y^2 - \left[\frac{d}{dx}(y^2)\right] x^2}{(y^2)^2}$$

$$= \frac{(2x)y^2 - \left[(2y) \cdot \frac{d}{dx}(y)\right] x^2}{y^4}$$

$$= \frac{2xy^2 - (2yy')x^2}{y^4}$$

$$= \frac{2xy - 2x^2 y'}{y^3}$$

$$= \frac{2xy - 2x^2 \left(\frac{x^2}{y^2}\right)}{y^3} \cdot \frac{y^2}{y^2}$$

$$= \frac{2xy^3 - 2x^4}{y^5}$$

$$= \frac{2x(y^3 - x^3)}{y^5}$$

$$= \frac{2x(-7)}{y^5}$$

$$= -\frac{14x}{y^5}$$